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ECE 204 *Numerical methods*

Approximating the solution to analytic equations

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1

Introduction

- In this topic, we will
    - Describe what will be covered in the next section
        - Approximating solutions to analytic equations
    - Describe differential, integral and integro-differential equations
    - Explain why we are focused on differential equations
    - Look at the approach we will use

2

2

1

Analytic equations

- In your calculus course, you have been introduced to two flavors of differential equations (DEs):
  - Ordinary differential equations (ODEs)
  - Partial differential equations (PDEs)
- Together with DEs, analytic equations can also contain integrals
  - Engineers focus on DEs, but all techniques are applicable to:
    - Differential equations
    - Integral equations
    - Integro-differential equations

3

3

Differential equations

- An ordinary derivative is the derivative of a function of one variable with respect to that variable

$$y^{(1)}(t) \qquad \frac{\mathrm{d}}{\mathrm{d}t}\,y(t)$$

- A partial derivative is a derivative of a function of two or more variables with respect to one of those variables while holding the other variables constant
  - Partial differentiation assumes no variable depends on the other variables

$$u_x(t, x, y) \qquad \frac{\partial}{\partial t}u(t, x, y) \qquad \frac{\partial}{\partial x_1}u(t, \mathbf{x})$$
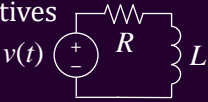
4

4

Analytic equations

- A differential equation involves an unknown function and its derivatives

$v(t)$, R, L

$$Li^{(1)}(t) + Ri(t) = v(t)$$

- An integral equation involves an unknown function and its integrals:

$v(t)$, R, C

$$Ri(t) + v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau = v(t)$$

- An integro-differential equation involves an unknown function and its integrals and derivatives:

$v(t)$, R, C, L

$$Li^{(1)}(t) + Ri(t) + v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau = v(t)$$

5

5

Tools

- By taking derivatives, integral and integro-differential equations can be converted to pure differential equations:

$$Li^{(1)}(t) + Ri(t) + v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau = v(t)$$

$$Li^{(2)}(t) + Ri^{(1)}(t) + \frac{1}{C}i(t) = v^{(1)}(t)$$

- Maxwell's equations can be written either as integral equations or partial differential equations

$$\oiint_{\partial R} \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_R \rho\,dV \qquad\qquad \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$$

  – To make life simple, engineers focus on differential equations

6

6

Analytic equations

- In this section, we will look at:
  - Approximating solutions to problems involving ordinary differential equations
  - Approximating solutions to problems involving partial differential equations

7

Approach

- In each case, we will be attempting to approximate a continuous and differentiable function
  - We cannot approximate such a function everywhere
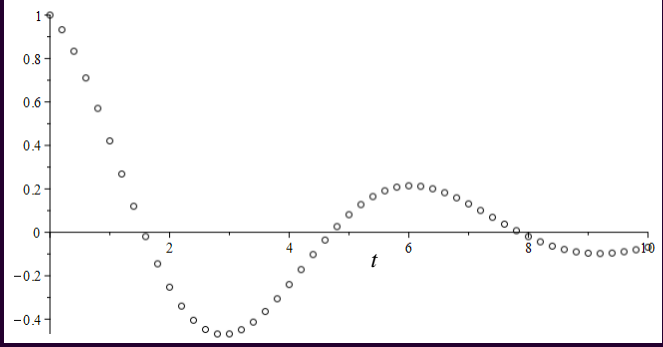 - Instead, we will, for example, approximate the function at specific points and we can interpolate values in between

8

Approach

- For example, if we are finding a function of one variable $y(t)$,
  we will approximate the function at points t_k and
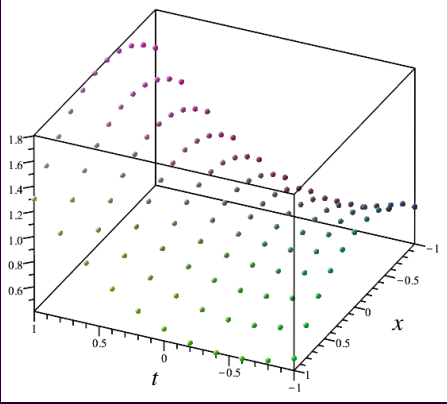 find approximations $y_k \approx y(t_k)$

9

9

Approach

- In finding a function of one two variables $u(t, x)$,
  we will approximate the function at points (t_ℓ, x_k) and
  find approximations $u_{\ell,k} \approx u(t_\ell, x_k)$

10

10

5

Summary

- Following this topic, you now
  - Have an overview of the ideas to be covered in this section
  - Are aware that there are differential, integral and intego-differential equations
  - Understand we will focus on ordinary and partial differential equations
  - Are aware of the approach we will use

11

11

References

[1]   https://en.wikipedia.org/wiki/System_of_linear_equations
[2]   https://en.wikipedia.org/wiki/Nonlinear_system

12

12

Acknowledgments

None so far.

13

13

Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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14

14

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15

15